Spiral plat avec courbes terminales externe et interne

Anisochronisme en position horizontale

Approximations de Haag

Caractéristiques du spiral

Référence :C:\Résonateur (TA)\Data\Bal_spiral plat (ex num).mcd(R)

Référence :C:\Résonateur (TA)\Data\Définition Atan.mcd(R)

Dimensions $ép = 0.03\,mm$ $ha = 0.15\,mm$ $S = 4.5\times 10^{-3}\,mm^2$ $TOL := 10^{-12}$

$d2_{sp} = 4.52\,mm$ $d_V := 1.1\cdot mm$ $d_B := 1.312\cdot d1_{sp}$ $p_{sp} = 0.135\,mm$ $n_{sp} := \dfrac{d2_{sp} - d_B}{2\cdot p_{sp}}$

$L := \pi\cdot\dfrac{n_{sp}}{2}\cdot\left(d2_{sp} + d_B\right)$ $L = 10.674\,cm$ $\psi_0 := 2\cdot\pi\cdot n_{sp}$ $\psi_0 = 4.102\times 10^3\,deg$

Position du point de raccordement sur le spiral $\alpha_A := \pi$ $r_A := 0.5\cdot d2_{sp}$ $z_A := r_A\cdot e^{i\cdot\alpha_A}$

Forme initiale du spiral

$a := \dfrac{p_{sp}}{2\cdot\pi}$ $r_s(\alpha) := r_A - a\cdot\left(\alpha - \alpha_A\right)$ $x_{0s}(\alpha) := r_s(\alpha)\cdot cos(\alpha)$ $y_{0s}(\alpha) := r_s(\alpha)\cdot sin(\alpha)$

$s(\alpha) := \dfrac{1}{2\cdot a}\cdot\left(r_A{}^2 - r_s(\alpha)^2\right)$ $s(\alpha) := r_A\cdot\left(\alpha - \alpha_A\right) - \dfrac{a}{2}\cdot\left(\alpha - \alpha_A\right)^2$

Référence :C:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$I_{33} := I_{f\_rect}(ép, ha)$

Courbes terminales

Courbe terminale externe

$r_{t1} := 0.8$ $r_{t1} := racine\left[\left(2\cdot r_{t1} - 1\right)^4 - 4\cdot\left(1 - r_{t1}\right)^4 - \pi^2\cdot r_{t1}{}^2\cdot\left(1 - r_{t1}\right)^2, r_{t1}\right]\cdot r_A$ $r_{t1} = 0.832\,r_A$

$r_{t2} := 2\cdot r_{t1} - r_A$ $r_{t2} = 0.665\,r_A$ $\beta_0 := arctan\left[\dfrac{\pi\cdot r_{t1}}{2\cdot\left(r_A - r_{t1}\right)}\right]$ $\beta_0 = 82.695\,deg$ $l_t := r_{t2}\cdot\beta_0 + \pi\cdot r_{t1}$

$X_{0t1}\left(\alpha_t\right) := r_A - r_{t1} + r_{t1}\cdot cos\left(\alpha_t\right)$ $Y_{0t1}\left(\alpha_t\right) := r_{t1}\cdot sin\left(\alpha_t\right)$ $X_{0t2}\left(\beta_t\right) := -r_{t2}\cdot cos\left(\beta_t\right)$ $Y_{0t2}\left(\beta_t\right) := -r_{t2}\cdot sin\left(\beta_t\right)$

Courbe terminale interne

$\alpha_B := mod\left(\psi_0 + \pi, 2\cdot\pi\right)$ $\alpha_B = 322.4\,deg$ $r_B := 0.5\cdot d_B$

$\beta := 121\cdot deg$ $\beta'_0 := racine\left[\beta\cdot\left(\sqrt{2}\cdot sin(\beta) - 1\right) + sin(\beta)\cdot cos(\beta), \beta\right]$ $\beta'_0 = 121.21\,deg$

$r_{t'} := \dfrac{r_B}{\sqrt{2}\cdot sin\left(\beta'_0\right)}$ $r_{t'} = 0.597\,mm$ $l_{t'} := r_{t'}\cdot 2\cdot\beta'_0$ $l_{t'} = 2.524\,mm$

$X_{0t'}\left(\alpha_{t'}\right) := r_B - r_{t'} + r_{t'}\cdot cos\left(\alpha_{t'}\right)$ $Y_{0t'}\left(\alpha_{t'}\right) := r_{t'}\cdot sin\left(\alpha_{t'}\right)$

Graphes

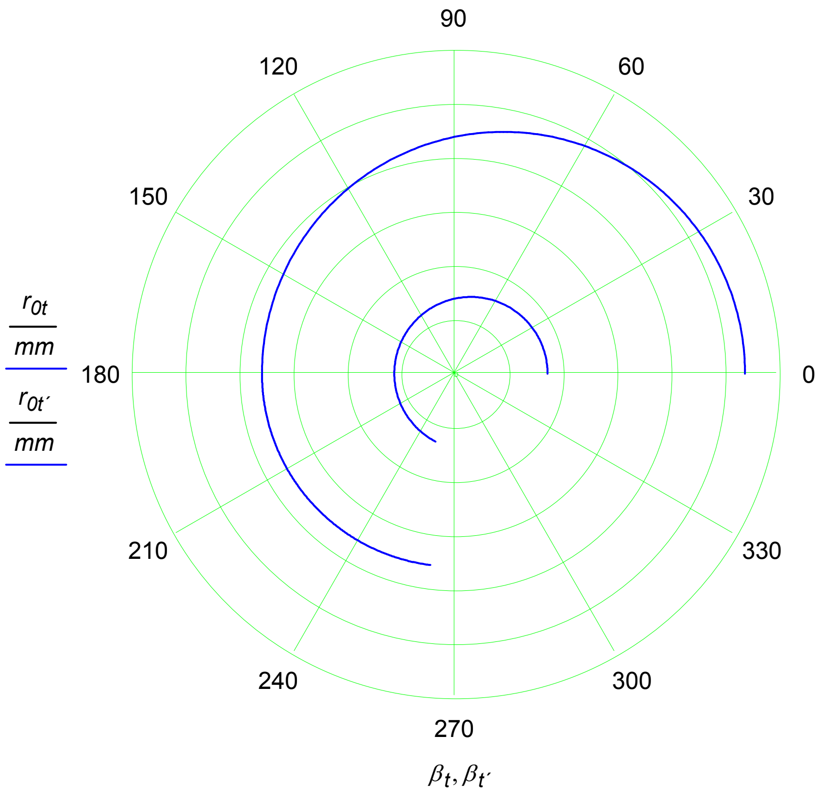
$n_t := 201$ $j := 0 .. n_t - 1$ $\Delta\alpha_t := \dfrac{\pi}{n_t - 1}$ $\alpha_{t_j} := j\cdot\Delta\alpha_t$ $X_{t_j} := X_{0t1}\left(\alpha_{t_j}\right)$ $Y_{t_j} := Y_{0t1}\left(\alpha_{t_j}\right)$

$\Delta\beta_t := \dfrac{\beta_0}{n_t - 1}$ $\beta_{t_j} := j\cdot\Delta\beta_t$ $X_{t2_j} := X_{0t2}\left(\beta_{t_j}\right)$ $Y_{t2_j} := Y_{0t2}\left(\beta_{t_j}\right)$ $X_t := pile\left(X_t, X_{t2}\right)$ $Y_t := pile\left(Y_t, Y_{t2}\right)$

$r_{0t} := \overrightarrow{\sqrt{X_t^2 + Y_t^2}}$ $\beta_t := \overrightarrow{Atan(X_t, Y_t)}$

$\Delta\alpha_{t'} := \dfrac{2 \cdot \beta'_0}{n_t - 1}$ $\alpha_{t'_j} := j \cdot \Delta\alpha_{t'}$ $X_{t'_j} := X_{0t'}\left(\alpha_{t'_j}\right)$ $Y_{t'_j} := Y_{0t'}\left(\alpha_{t'_j}\right)$ $r_{0t'} := \overrightarrow{\sqrt{X_{t'}^2 + Y_{t'}^2}}$

$\beta_{t'} := \overrightarrow{Atan(X_{t'}, Y_{t'})}$



Paramètres de la courbe terminale externe

$$X_1 := \frac{1}{r_A^2} \cdot \left(\int_0^{\pi} X_{0t1}(\alpha) \cdot r_{t1} \, d\alpha + \int_0^{\beta_0} X_{0t2}(\beta) \cdot r_{t2} \, d\beta \right) \qquad X_1 = 0$$

$$Y_1 := \frac{1}{r_A^2} \cdot \left(\int_0^{\pi} Y_{0t1}(\alpha) \cdot r_{t1} \, d\alpha + \int_0^{\beta_0} Y_{0t2}(\beta) \cdot r_{t2} \, d\beta \right) - 1 \qquad Y_1 = 0$$

$\rho_1 := \sqrt{X_1^2 + Y_1^2}$ $\varphi_1 := Atan(X_1, Y_1)$ $\boxed{\rho_1 = 0}$ $\boxed{\varphi_1 = 270\ deg}$

$$X_2 := \frac{1}{r_A^3} \cdot \left[\int_0^{\pi} r_{t1} \cdot \alpha \cdot X_{0t1}(\alpha) \cdot r_{t1} \, d\alpha + \int_0^{\beta_0} (r_{t1} \cdot \pi + r_{t2} \cdot \beta) \cdot X_{0t2}(\beta) \cdot r_{t2} \, d\beta \right] + 1$$

$$Y_2 := \frac{1}{r_A^3} \cdot \left[\int_0^{\pi} r_{t1} \cdot \alpha \cdot Y_{0t1}(\alpha) \cdot r_{t1} \, d\alpha + \int_0^{\beta_0} (r_{t1} \cdot \pi + r_{t2} \cdot \beta) \cdot Y_{0t2}(\beta) \cdot r_{t2} \, d\beta \right]$$

$\rho_2 := \sqrt{X_2^2 + Y_2^2}$ $\varphi_2 := Atan(X_2, Y_2)$ $\boxed{\rho_2 = 1.055}$ $\boxed{\varphi_2 = 147.579\ deg}$

Paramètres de la courbe terminale interne

$$Z_{0t'}(\alpha) := X_{0t'}(\alpha) + i\cdot Y_{0t'}(\alpha)$$

$$Z'_1 := \frac{1}{r_B^2}\cdot\int_0^{2\cdot\beta'_0} Z_{0t'}(\alpha)\cdot r_{t'}\,d\alpha - i \qquad \rho'_1 := \left|Z'_1\right| \qquad \varphi'_1 := arg\left(Z'_1\right)$$

$$\boxed{\rho'_1 = 0} \qquad \boxed{\varphi'_1 = -25.264\,deg}$$

$$Z'_2 := \frac{1}{r_B^3}\cdot\int_0^{2\cdot\beta'_0} r_{t'}\cdot\alpha\cdot Z_{0t'}(\alpha)\cdot r_{t'}\,d\alpha + 1 \qquad \rho'_2 := \left|Z'_2\right| \qquad \varphi'_2 := arg\left(Z'_2\right)$$

$$\boxed{\rho'_2 = 1.074} \qquad \boxed{\varphi'_2 = 145.651\,deg}$$

Déplacement de la virole libre

$$\mathbf{OA} := r_A\cdot e^{i\cdot\pi} \qquad \mathbf{OB} := r_B\cdot e^{i\cdot(\pi+\psi_0)} \qquad L_t := l_t + L + l_{t'}$$

$$\mathbf{w_A}(\rho_1,\theta) := \frac{\theta}{L_t}\cdot\left[i\cdot\left(r_A\cdot\rho_1\cdot e^{-i\cdot\varphi_1} + 2\cdot a\right) + \frac{\theta}{L_t}\cdot r_A^2\cdot\rho_2\cdot e^{-i\cdot\varphi_2}\right]\cdot exp\left(i\cdot\theta\cdot\frac{l_t}{L_t}\right)\cdot\mathbf{OA}$$

$$\mathbf{w_B}(\rho'_1,\theta) := \frac{\theta}{L_t}\cdot\left[i\cdot\left(r_B\cdot\rho'_1\cdot e^{i\cdot\varphi'_1} - 2\cdot a\right) - \frac{\theta}{L_t}\cdot r_B^2\cdot\rho'_2\cdot e^{i\cdot\varphi'_2}\right]\cdot exp\left(i\cdot\theta\cdot\frac{l_t+L}{L_t}\right)\cdot\mathbf{OB}$$

$$\mathbf{w}(\rho_1,\rho'_1,\theta) := \mathbf{w_A}(\rho_1,\theta) + \mathbf{w_B}(\rho'_1,\theta) \qquad \mathbf{w}(\rho_1,\rho'_1,\theta_0) = 0.012 + 0.012i\,mm$$

En éliminant les termes de second ordre

$$\mathbf{w_{aA}}(\theta) := \frac{\theta}{L_t}\cdot\left[i\cdot\left(r_A\cdot\rho_1\cdot e^{-i\cdot\varphi_1} + 2\cdot a\right) + \frac{\theta}{L_t}\cdot r_A^2\cdot\rho_2\cdot e^{-i\cdot\varphi_2}\right]\cdot\mathbf{OA}$$

$$\mathbf{w_{aB}}(\theta) := \frac{\theta}{L_t}\cdot\left[i\cdot\left(r_B\cdot\rho'_1\cdot e^{i\cdot\varphi'_1} - 2\cdot a\right) - \frac{\theta}{L_t}\cdot r_B^2\cdot\rho'_2\cdot e^{i\cdot\varphi'_2}\right]\cdot\mathbf{OB}\cdot e^{i\cdot\theta} \qquad \mathbf{w_{ah}}(\theta) := \mathbf{w_{aA}}(\theta) + \mathbf{w_{aB}}(\theta)$$

$$\mathbf{w_{ah}}(\theta_0) = 0.015 + 7.189i\times 10^{-3}\,mm$$

Réaction sur le pivot de balancier

$$\sigma 2 := \frac{r_A^2 + r_B^2}{2} \qquad \sigma 2 = 2.814\,mm^2 \qquad \mathbf{F}(\theta) := 2\cdot\frac{E\cdot I_{33}}{L\cdot\sigma 2}\cdot\mathbf{w_{ah}}(\theta) \qquad \left|\mathbf{F}(\theta_0)\right| = 7.812\times 10^{-6}\,N$$

Perturbation de période

$$X_w(\theta) := \frac{\left(\left|\mathbf{w}(\rho_1,\rho'_1,\theta)\right|\right)^2}{\sigma 2} \qquad \gamma_w(\theta) := \frac{d}{d\theta}X_w(\theta) \qquad \delta_w(\theta_0) := \frac{-1}{2\cdot\pi\cdot\theta_0}\cdot\int_0^{2\cdot\pi}\gamma_w(\theta_0\cdot cos(\varphi))\cdot cos(\varphi)\,d\varphi$$

$$\mu_w(\theta_0) := -86400\cdot\delta_w(\theta_0) \qquad \boxed{\mu_w(\theta_0) = 0.792} \qquad \boxed{\mu_w(180\cdot deg) = 0.369}$$

$$X_{ah}(\theta) := \frac{\left(\left|\mathbf{w_{ah}}(\theta)\right|\right)^2}{\sigma 2} \qquad \gamma_{ah}(\theta) := \frac{d}{d\theta}X_{ah}(\theta) \qquad \delta_{ah}(\theta_0) := \frac{-1}{2\cdot\pi\cdot\theta_0}\cdot\int_0^{2\cdot\pi}\gamma_{ah}(\theta_0\cdot cos(\varphi))\cdot cos(\varphi)\,d\varphi$$

$$\mu_{ah}(\theta_0) := -86400\cdot\delta_{ah}(\theta_0) \qquad \boxed{\mu_{ah}(\theta_0) = 0.796} \qquad \boxed{\mu_{ah}(180\cdot deg) = 0.363}$$

$$A := \frac{1}{L_t^2}\cdot\left[r_A^4\cdot\rho_1^2 + r_B^4\cdot\rho'_1{}^2 + 4\cdot a\cdot\left(r_A^3\cdot\rho_1\cdot\cos(\varphi_1) - r_B^3\cdot\rho'_1\cdot\cos(\varphi'_1)\right) + 4\cdot a^2\cdot\left(r_A^2 + r_B^2\right)\right]$$

$$B := \frac{3}{2\cdot L_t^4}\cdot\left(r_A^6\cdot\rho_2^2 + r_B^6\cdot\rho'_2{}^2\right) \qquad C1 := r_A^2\cdot r_B^2\cdot\rho_1\cdot\rho'_1\cdot\cos(\psi_0 + \varphi_1 + \varphi'_1)$$

$$C := \frac{2}{L_t^2}\cdot\left[C1 + 2\cdot a\cdot r_A\cdot r_B\cdot\left(r_B\cdot\rho'_1\cdot\cos(\psi_0 + \varphi'_1) - r_A\cdot\rho_1\cdot\cos(\psi_0 + \varphi_1) - 2\cdot a\cdot\cos(\psi_0)\right)\right]$$

$$D1 := r_A\cdot r_B^2\cdot\rho_1\cdot\rho'_2\cdot\cos(\psi_0 + \varphi_1 + \varphi'_2) + r_A^2\cdot r_B\cdot\rho'_1\cdot\rho_2\cdot\cos(\psi_0 + \varphi'_1 + \varphi_2)$$

$$D := \frac{2\cdot r_A\cdot r_B}{L_t^3}\cdot\left[D1 + 2\cdot a\cdot\left(r_B^2\cdot\rho'_2\cdot\cos(\psi_0 + \varphi'_2) - r_A^2\cdot\rho_2\cdot\cos(\psi_0 + \varphi_2)\right)\right]$$

$$K := \frac{2}{L_t^4}\cdot r_A^3\cdot r_B^3\cdot\rho_2\cdot\rho'_2\cdot\cos(\psi_0 + \varphi_2 + \varphi'_2)$$

$$F(x) := J0(x) - x\cdot J1(x) \qquad H(x) := x\cdot\left(1 + x^2\right)\cdot J1(x) - 2\cdot x^2\cdot J0(x) \qquad G(x) := -x\cdot(J1(x) + x\cdot J0(x))$$

$$\delta_{ah}(\theta_0) := \frac{-1}{\sigma 2}\cdot\left(A + B\cdot\theta_0^2 + C\cdot F(\theta_0) + D\cdot G(\theta_0) + K\cdot H(\theta_0)\right)$$

$$\mu_{ah}(\theta_0) := -86400\cdot\delta_{ah}(\theta_0) \qquad \boxed{\mu_{ah}(\theta_0) = 0.796} \qquad \boxed{\mu_{ah}(180\cdot deg) = 0.363}$$

Spiral muni de courbes Phillips

$$\mathbf{w_{Ph}}(\theta) := \mathbf{w}(0, 0, \theta) \qquad \mathbf{w_{Ph}}(\theta_0) = 0.012 + 0.012i\,mm$$

$$X_{Ph}(\theta) := \frac{\left(\left|\mathbf{w_{Ph}}(\theta)\right|\right)^2}{\sigma 2} \qquad \gamma_{Ph}(\theta) := \frac{d}{d\theta}X_{Ph}(\theta) \qquad \delta_{Ph}(\theta_0) := \frac{-1}{2\cdot\pi\cdot\theta_0}\cdot\int_0^{2\cdot\pi}\gamma_{Ph}(\theta_0\cdot\cos(\varphi))\cdot\cos(\varphi)\,d\varphi$$

$$\mu_{Ph}(\theta_0) := -86400\cdot\delta_{Ph}(\theta_0) \qquad \boxed{\mu_{Ph}(\theta_0) = 0.792} \qquad \boxed{\mu_{Ph}(180\cdot deg) = 0.369}$$

Approximation de Haag

$$\delta_r(\theta_0) := \frac{4\cdot r_A^6}{\left(r_A^2 + r_B^2\right)\cdot L_t^4}\cdot\frac{r_B}{r_A}\cdot\left(\cos(\psi_0)\cdot F(\theta_0) + \rho_2\cdot\cos(\psi_0 + \varphi_2)\cdot G(\theta_0)\right) \qquad \delta_r(\theta_0) = 2.816\times 10^{-7}$$

$$\delta_{aPh}(\theta_0) := \frac{-8\cdot(r_A - r_B)^2}{L_t^2\cdot\psi_0^2} - \frac{3\cdot r_A^6}{\left(r_A^2 + r_B^2\right)\cdot L_t^4}\cdot\rho_2^2\cdot\theta_0^2 + \delta_r(\theta_0) \qquad \delta_{aPh}(\theta_0) = -9.248\times 10^{-6}$$

$$\mu_{aPh}(\theta_0) := -86400\cdot\delta_{aPh}(\theta_0) \qquad \boxed{\mu_{aPh}(\theta_0) = 0.799} \qquad \boxed{\mu_{aPh}(180\cdot deg) = 0.355}$$

$\theta_m := 100 \cdot deg, 105 \cdot deg .. 360 \cdot deg$